AN EAR FOR PYTHAGOREAN HARMONICS: MATHEMATICAL PROCESSING IN BRAIN MODELS BUILT FROM "TIMING DEVICES"

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ABSTRACT: A system of "timing devices," part of an alternative approach to neuroscience, is presented through development of a proposed application, namely, an "Ear," that specifically detects "Pythagorean Harmonics," signaling when inputs (two pure tones) have frequencies in a ratio of simple whole numbers, e.g., 3/2, which identifies the "dominant" in musical harmony.

Contents

For greater simplicity, material inside{...}can be disregarded.

§ 1 “Timing Devices,” an alternative approach to neuroscience.

“Timing Devices” are part of an alternative approach to neuroscience. Proposed timing devices are simplified models of brain cells (neurons) and make up a “kit of parts” that can be interconnected to form assemblies that function like parts of brains. Similarly, an electronics researcher interconnects transistors, resistors, capacitors, etc. to form electronic circuits. As shown herein, timing device assemblies can perform processes with a mathematical character. All designs are idealized and conceptual; there are no plans for working models.

A signal in an assembly of timing devices consist of “pulses.” A pulse is a uniform packet of energy; all pulses are “the same.” A pulse is an idealized “action potential” or “spike” in neurons. Here a pulse occurs “instantaneously” – the duration of a pulse is much smaller than that of any other timing interval. {More rigorously, a pulse resembles a Dirac delta function.}

a. ***The primal timing device.*** The “primal timing device” models a very simple brain cell (neuron) and is a point of origin for development of more complex timing devices.

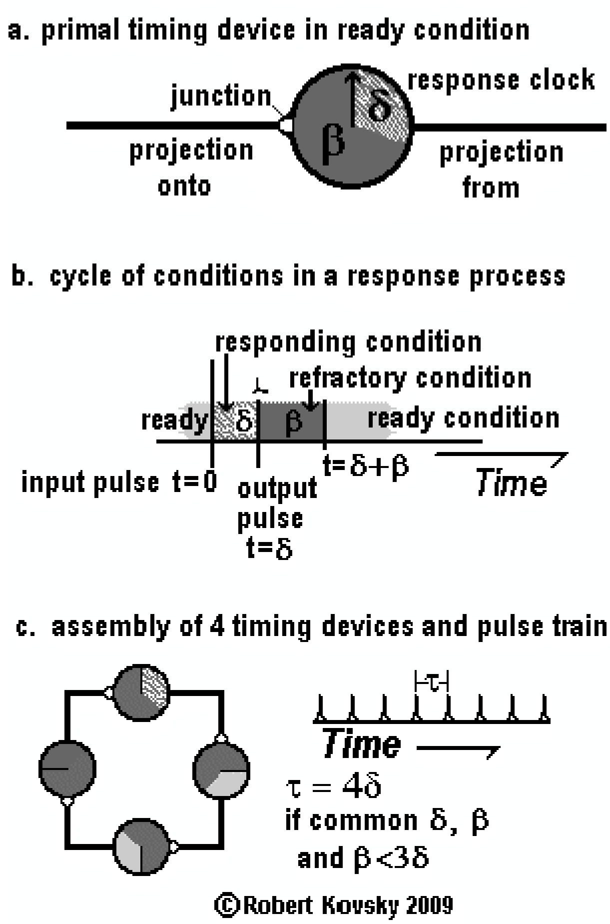


Figure 1: Primal Timing Device

The chief part of a primal timing device is a “response clock” that works like a stopwatch used in sports contests and that is shown in Figure 1.a as a circular clock face. Two “projections” connect the response clock to other timing devices. A “projection from” carries pulses away from the timing device. A “projection onto” carries pulses to the timing device. A “junction” connects a “projection onto” to the response clock and controls the “response process” that runs in the device.

A primal timing device has a “ready condition,” a “responding condition” and a “refractory condition.” The device responds to an incident input pulse only when it is in the ready condition. Suppose that, as shown in Figure 1.b, an input pulse reaches a ready device at time $t = 0$. The response clock starts and the device enters into the responding condition. At $t = \delta$, the device emits a pulse on the projection from; and the device enters into the refractory condition, which continues until $t = \delta + \beta$. δ is “the response period” and β is “the refractory period.” The system operator or system operations can vary δ and β. In a steady “pulse train” (see Figure 4.c), the period between pulses, τ, is fixed and $\tau = (\beta + \gamma + \delta)$, where γ is the “readiness period.”

Figure 1.c shows an assembly of four timing devices operating as a generator of a pulse train. A "pulse train" in a timing device assembly has functional parallels to a signal in an electronic circuit of transistors, etc. As the "voltage" can vary in an electronic circuit, so does τ, the period between pulses of a timing device engaged in activity. The only quantity that varies in the class of pulse trains is the period. {More complex signals are "pulse bundles" – see Figure 13.} The pulse train is maintained as shown in Figure 1.c while τ, β, δ and γ are in suitable relations, e.g., while $\tau = 4\delta$, $\beta = 1.1\delta$ and $\gamma = 1.9\delta$.

b. ***Complex timing devices.*** The timing devices system supports free invention. Changes and improvements to timing devices are made piecewise, starting with the primal timing device. As a result of developments shown here, complex timing device can have multiple "projections onto" and/or multiple "projections from." Pulses through different classes of junctions can control different aspects of operation. More complex systems of clocks are used. Although not used here, a "bursting timing device," suggested by biological neurons, can emit numerous pulses when triggered by a single incident pulse. Different kinds of timing devices can be put together to make up various useful assemblies.

This essay presents useful assemblies of timing devices that incorporate certain complex timing devices, which are all described here, in the nature of a "packing list" of a "kit of parts." Subsequent sections show elementary hookups that can be set into a larger developmental context, leading toward models of muscular movement, as in walking and dance.

Figure 2 shows initial development of the primal timing device, through multiplication of projections. For purposes of this and further figures, a simple box is substituted for the round clock face of Figure 1. When needed, "timing intervals" – e.g., β, γ, δ – are stated symbolically.

In Figure 2.a, multiple "projections from" carry identical signals away from the timing device. In this essay, the "equal-output rule" is always followed when there are multiple projections from.

Figure 2.b shows operation of multiple "projections onto" a timing device that is governed by the "one-pulse trigger rule," namely, that "one pulse" incident onto a ready timing device through ***any*** projection onto suffices to trigger the response process and start the response clock. {The one-trigger rule resembles the logical "or."}

Figure 2: multiple projections from and projections onto

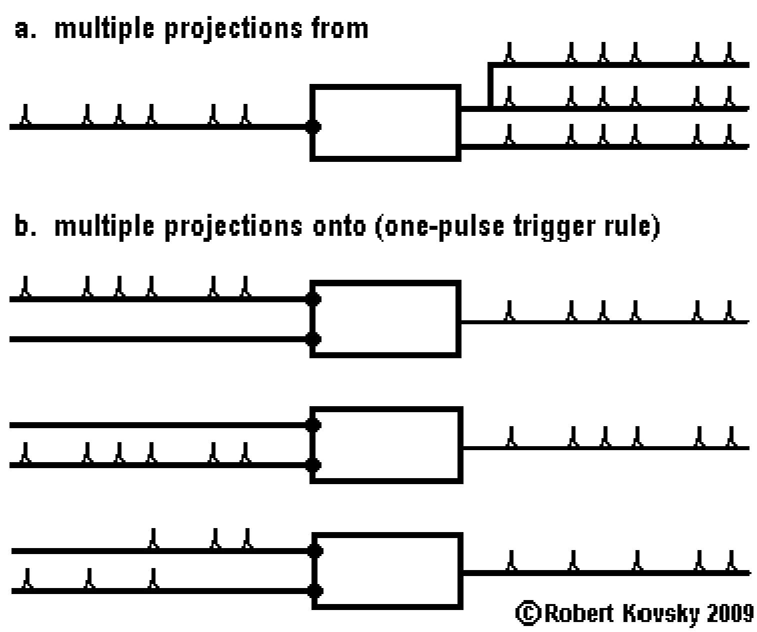


Figure 3 shows operations of a complex timing device governed by the "two-pulse trigger rule:" two input pulses are required to trigger the response process and start the response clock; and both pulses must arrive at the timing device within a defined timing interval, ξ. It doesn't matter which pulse arrives first. {The two-trigger rule resembles the logical "and."}

Suppose, as shown in Figure 3.a, one pulse arrives at $t=t_a$ and another pulse arrives at $t=t_b$. If the absolute value of the difference in time of arrivals is less than ξ, i.e., if $| t_b - t_a | < \xi$, then the response process will be triggered. On the other hand, if $| t_b - t_a | > \xi$, there will be no response.

Figure 3: two-pulse trigger rule

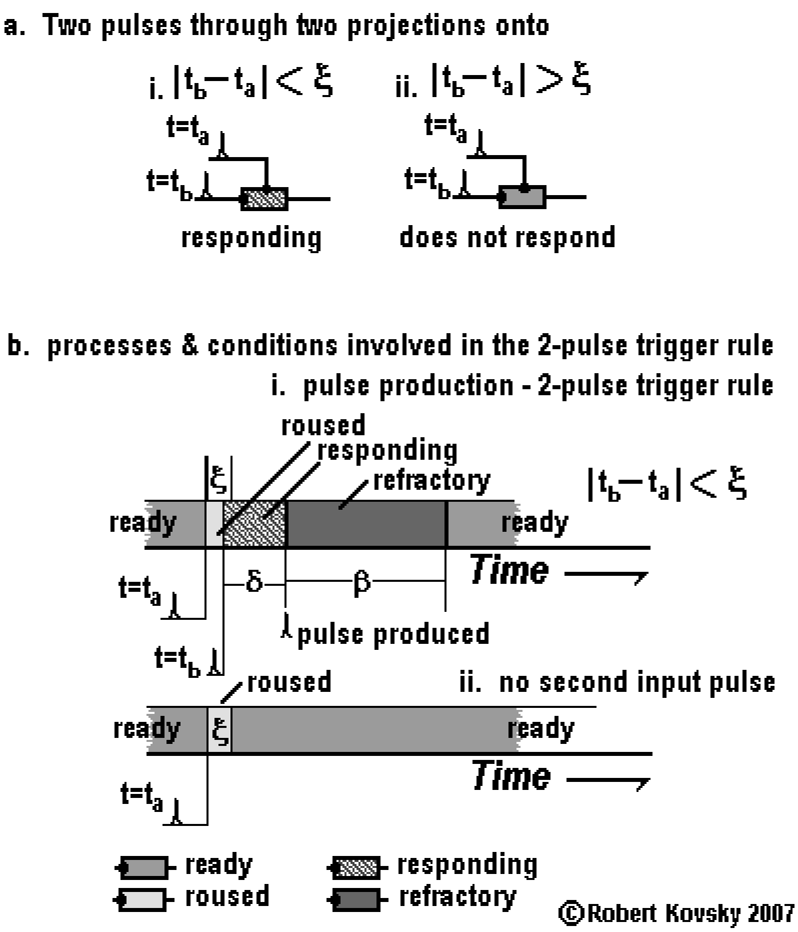


Figure 3.b shows the processes and conditions involved in the two-pulse trigger rule in greater detail. For purposes here, a new condition is introduced, the ***roused condition***. If a timing device governed by the two-pulse trigger rule is in the ready condition and a pulse arrives, the condition changes to the roused condition. If, while the timing device is in the roused condition, a second pulse arrives, the response clock starts and the condition changes to the responding condition. (See Figure 3.b.i.) If the roused condition continues for the full period of ξ without the arrival of a second pulse, the roused condition terminates and the timing device returns to the ready condition. (See Figure 3.b.ii.)

In a timing device governed by the two-pulse trigger rule, there is an additional clock, the ***rousal clock*** that runs while the timing device is in the roused condition. The rousal clock is initially stopped (like the response clock). If a pulse arrives when the timing device is in the ready condition, the rousal clock is started. Unless a second pulse arrives while the rousal clock is running, the rousal clock terminates after the period of time denoted by ξ and the timing device returns to the ready condition. If a second pulse arrives while the rousal clock is running, that arrival triggers the response process and starts the response clock.

Figure 4 shows another kind of complexity, where two junctions belong to different classes. As illustrated in Figure 4.a, a class 1 junction is the kind of junction used in the primal timing device, namely, a pulse onto it triggers the response process. Pulses onto a class 2 junction perform different functions: they modify the operational settings of the timing device onto which they are incident. E.g., they modify the δ of that timing device; or they "open a gate" or "close a gate" controlling pulse transmission. Such modifications are called "modulations."

Figure 4.a shows "continuous modulation of δ," suitable for a sensory input. The response period, δ, is a variable that depends on the input σ. The value of δ at the instant of trigger becomes the response period for that cycle. A stronger input results in a longer response period, δ, leading to a slower response process. The increased period measures the strength of the modulatory (sensory) input.

Figure 4.b shows "pulsational modulation of δ," suitable for controlling device operations. A modulation pulse incident upon the timing device through a class 2 junction changes the response period, δ, for a specified period of time, λ.

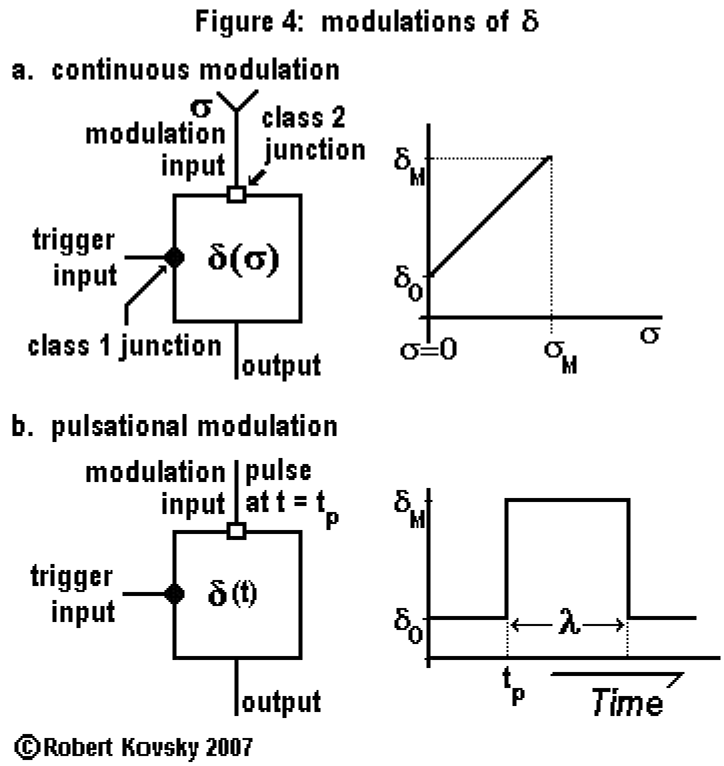


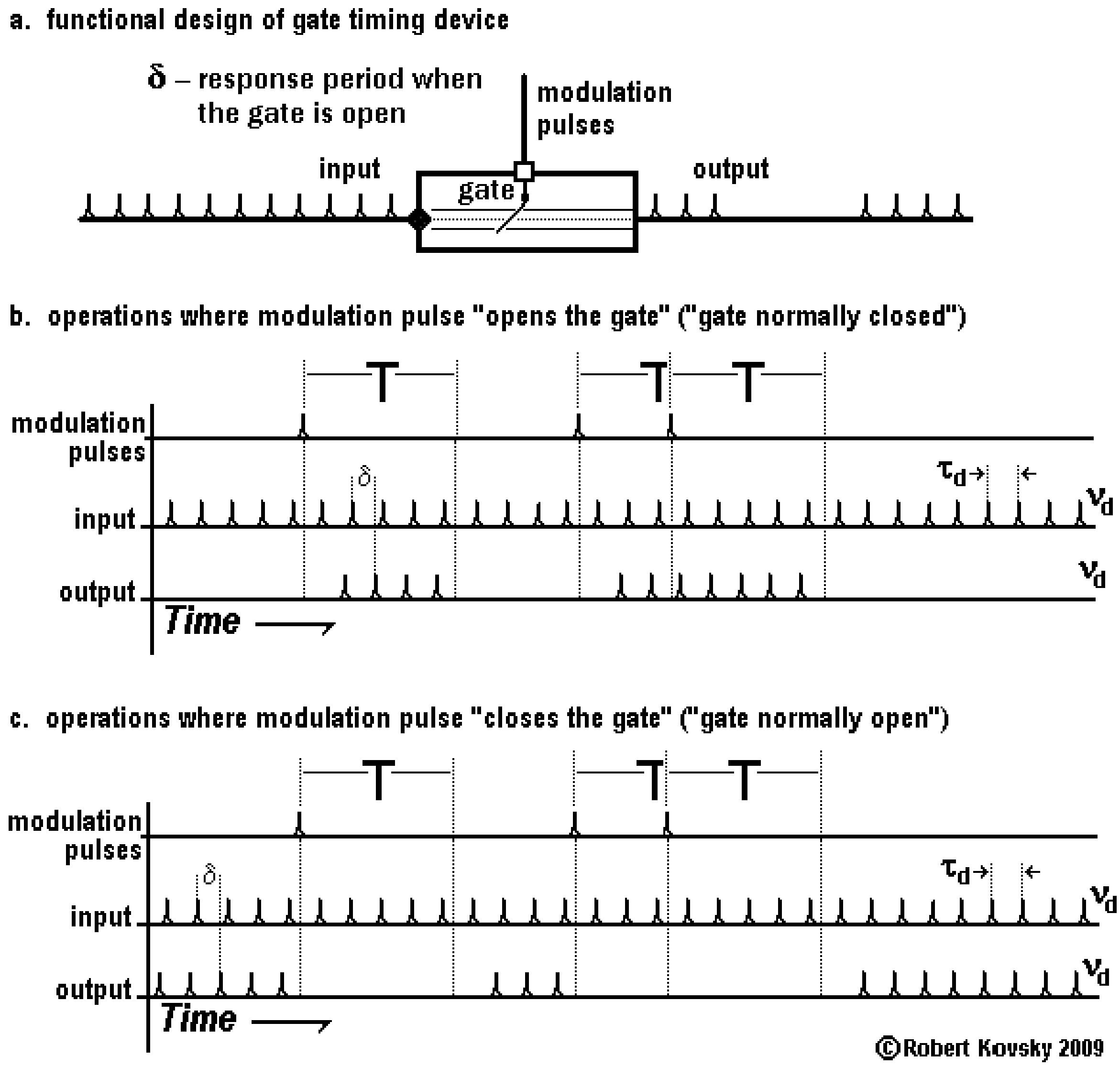
Figure 5 shows "gate timing devices" that have functional parallels to traditional electrical and electronic circuit components, e.g., with parallels to electrical relays, vacuum tube triodes and "npn" transistors, but with new kinds of signals and operations.

Figure 5.a shows the functional design of a gate timing device, where an input signal (e.g., the pulse train shown) generates an identical, delayed output signal when "the gate is open" but is silent when "the gate is closed." Modulation pulses control the opening (or closing) of the gate.

Figure 5.b shows operations of a gate timing device where a modulation pulse "opens the gate" for a time period T, leading to output for such time period, but only silence otherwise.

Figures 5.c shows operations of a gate timing device where a modulation pulse "closes the gate" and leads to silence for a time period T, although producing output otherwise.

Figure 5: gate timing devices



§ 2 Operations of simple assemblies: strings of timing devices with selectors.

Figure 6 shows a pulse passing through a string of timing devices. Time measurements are stated by reference to a "laboratory clock."

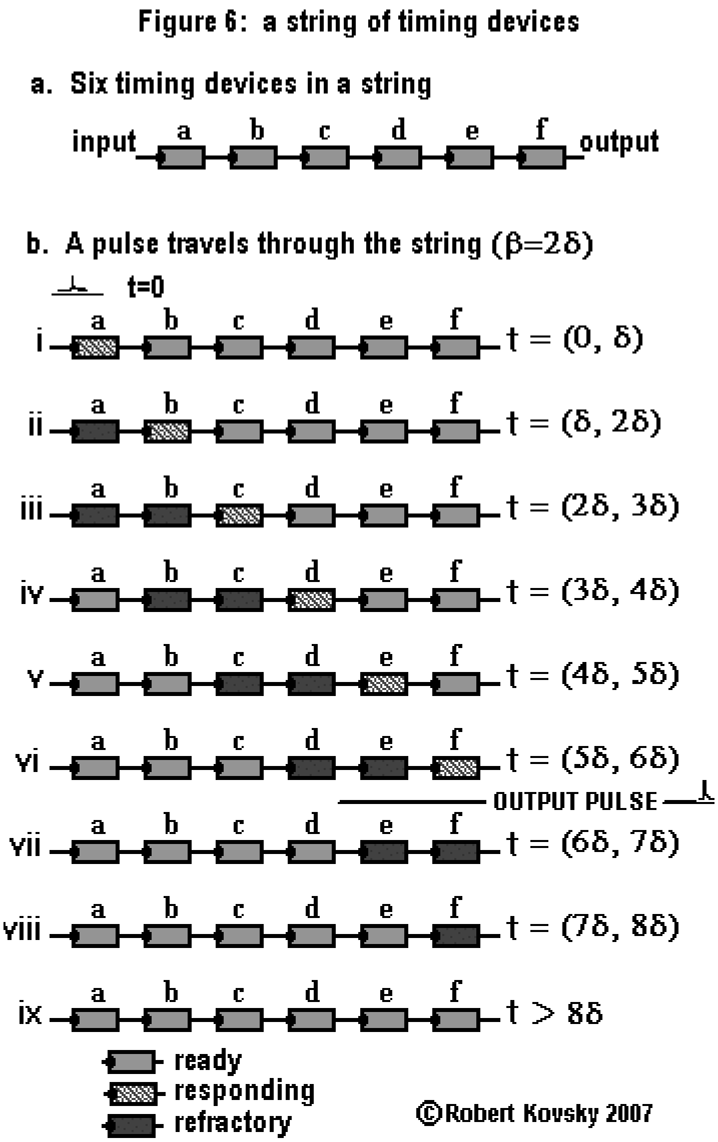


Figure 6.a shows six primal timing devices joined to make up a string. The only input to the assembly is directed onto timing device a. The only output is the projection from timing device f.

In Figure 6.a, all six timing devices are in the ready condition. Each can be triggered; but an input from outside is needed to initiate activity. The assembly is ready and waiting.

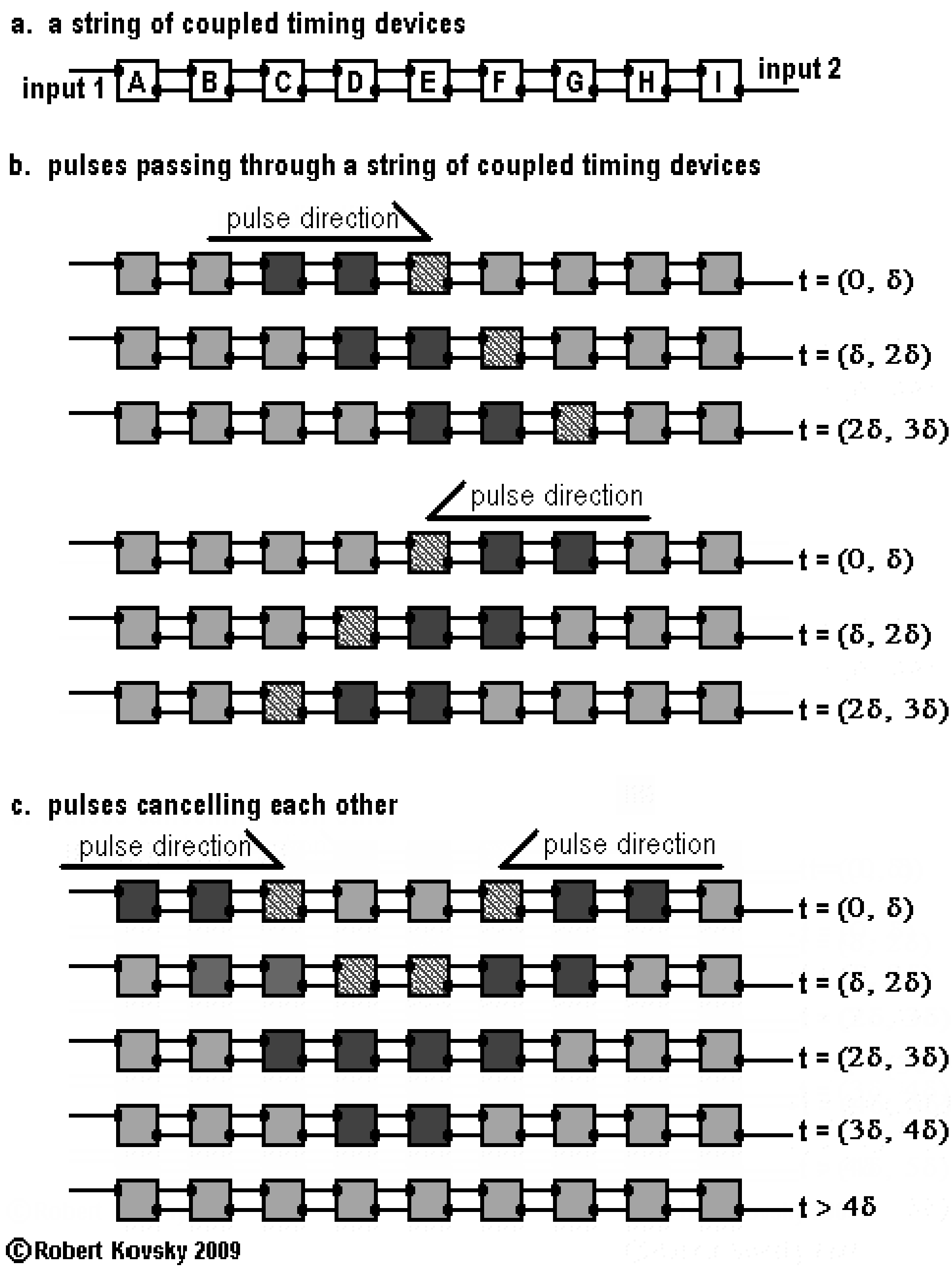
In Figure 6.b, all timing devices have a common response period δ and a common refractory period β, where β=2δ. Activity starts with an input pulse onto timing device a at time t=0. Figure 6.b.i shows conditions from just after t=0, when timing device a begins to respond, until just before t=δ, a period of time denoted by (0, δ). As time passes from t<δ to t>δ, timing device a produces an output pulse and changes to the refractory condition. The output pulse from timing device a becomes an input pulse onto timing device b that triggers that device, starting its response clock and changing its condition to "responding."

Figure 6.b.ii. shows the conditions between δ and 2δ. As t<2δ changes to t>2δ, timing device b produces an output pulse and changes to the refractory condition. The pulse produced by timing device b triggers the response process in timing device c and starts its clock, establishing conditions shown in Figure 3.b.iii. Timing device a remains in the refractory condition until t=3δ (=δ+β), when it again becomes ready.

As time evolves to and beyond t=3δ, each timing device successively reproduces the activity of its predecessor in the string; and the activity of each successive timing device is displaced in time by δ.   In effect, δ is the "clock tick" that marks changes in conditions in the string. The pulse passes through the string and emerges as an output pulse at t=6δ, shown in Figure 6.b. After t=8δ, the string is again ready and waiting.

Figure 7.a shows a string of "coupled timing devices." Each device is connected to each of its nearest neighbors by both a projection from and a projection onto. The "one-pulse rule" governs all timing devices. As shown in Figure 7.b, pulses can pass through such a string in either direction. If a pulse traveling in one direction meets a pulse traveling in the other direction, they "cancel" each other, as shown in Figure 7.c.

Figure 7: pulses in a string of coupled timing devices



Cancellation occurs in every case when pulses are directed against each other, including those where changes in timing device conditions are not synchronized as they are in Figure 7.c.

For proper operation of a string of coupled timing devices, β must be larger than δ; otherwise, a timing device whose discharge triggered a second timing device will itself by triggered when the second timing device discharges. That is, $\beta > \delta$ is a constraint on operations. β need be greater than δ by only a tiny amount for proper operations to proceed. If, however, β is less than δ, even if by only a tiny amount, proper operations are impossible: the device assembly discharges in a fixed and useless way. For purposes of easy imagery, $\beta = 2\delta$ in Figure 7 and in other Figures; but such an integral relation (2:1) is not requisite for any operation.

In general, a device assembly can function properly only when each timing interval is set to a value within "a range of values," usually stated as constraints. A full definition of an assembly includes specifying the limit points of ranges of values, describing failures of proper operations outside the limit points and showing that failure is avoided within the limit points.

{To support a train of pulses in a string of coupled timing devices, the cycle of activity must include a readiness period, γ, greater than 0; but $\gamma>0$ need be no more than a tiny amount. The smallest cycle period, $\tau_{MIN} = \delta + \beta_{MIN} + \gamma_{MIN} = 2\delta$ + a tiny bit more (to cover both $\beta_{MIN} > \delta$ and $\gamma_{MIN} > 0$), which is symbolized as $\tau_{MIN} = 2\delta^{+}$. If $\delta = 10^{-3}$ sec., the maximum workable frequency for the string is a bit under 500 Hz; if $\delta = 10^{-4}$ sec., the maximum workable frequency for the string is just below 5000 Hz. Attempts to operate the string at frequencies close to but above the cutoff will result in output pulse trains with exactly half the frequency of the input. In other words, operating just outside the range of τ for perfect transmission, the string becomes a "frequency divider," alternatively passing and failing to pass a pulse.}

Figure 8: design of selector

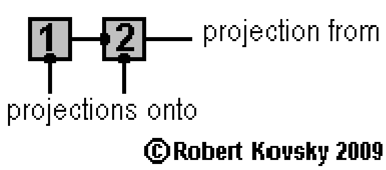


Figure 8 shows the design of a "selector." A selector is a functional assembly of timing devices that is attached to another assembly and that detects a pulse traveling in one direction in that assembly but not the other. The selector exists as its own independent assembly prior to attachment to other assemblies, (e.g. as shown below).

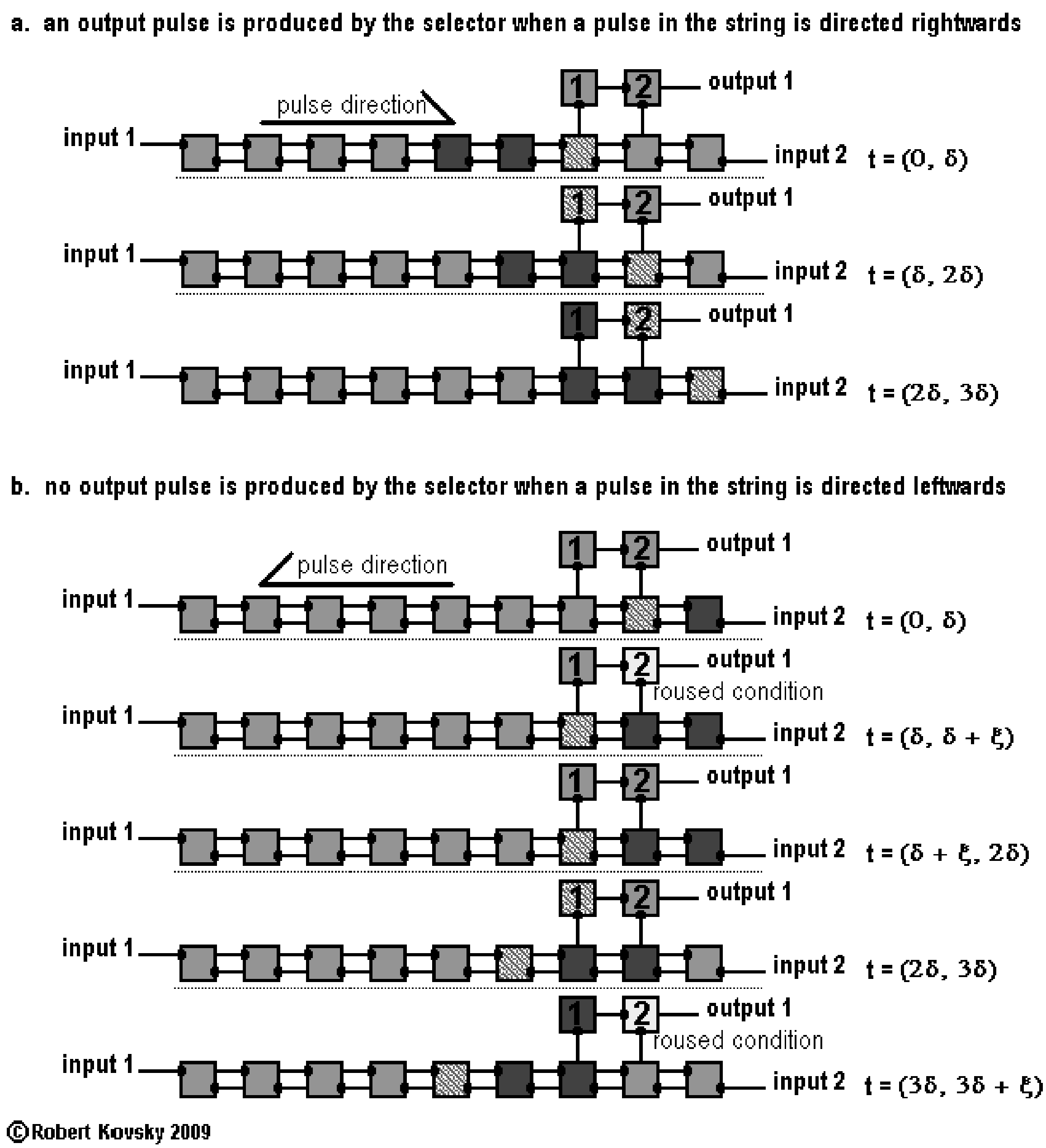
In Figure 8, the timing device labeled "1" operates according to the one-pulse trigger rule; the timing device labeled "2" operates according to the two-pulse trigger rule (see Figure 3 and surrounding text). The operating principle of the selector is that the "projections onto" must be activated in a 1, 2 order, one δ apart, for a pulse to appear on the "projection from." No output pulse appears if the projections onto are activated in a 2, 1 order.

{The δ and β of timing devices in the selector are adjusted to match those of the assembly to which the selector is attached. The ξ of timing device 2 is much less than δ or β. The value of ξ is of secondary importance here because designs avoid "edges" where variations in ξ might be consequential. Generally, you can assume $\xi = 0.1\delta$. Some Figures do not annotate ξ.}

Figure 9 shows a selector in operation. In Figure 9.a, the selector is "counting" a pulse that is directed rightwards. That is, at time t= 3δ, timing device 2 will discharge and produce a pulse through output 1. Output 1 is connected to a "counter" that maintains a "count," which increases by 1 each time a pulse passes through. As shown in Figure 9.b, the selector does not count pulses directed leftwards. In such case, the "roused condition" in timing device 2 only lasts for ξ, much less than δ, and the roused condition does not change to a responding condition.

{Although treated as a state variable here, the "count" does not adequately measure a pulse train (which may be irregular) and the "count" has only limited applicability.}

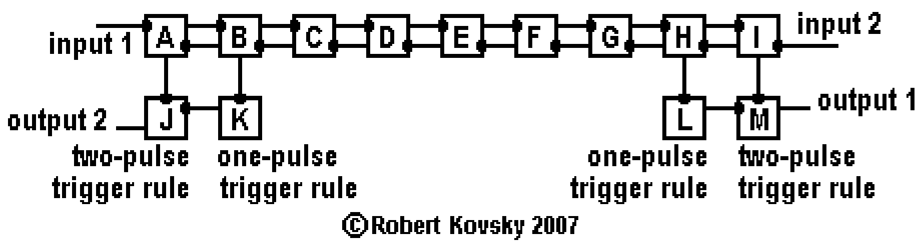
Figure 9: the selector counts pulses directed rightwards and disregards pulses directed leftwards



§ 3 A crude frequency subtractor, using a string of coupled timing devices, applied to maintain alignment (balance) in an engineered organism.

Fig. 10 shows an assembly of timing devices where two inputs and two outputs are attached to a string of coupled timing devices. This design was Fig. 25 in *Selecting and Controlling Action With Networked Timing Devices* ("*Timing Devices*"), an earlier version of this essay, which set forth technical details of this system.

Figure 10: string of coupled timing devices with two inputs and attached outputs

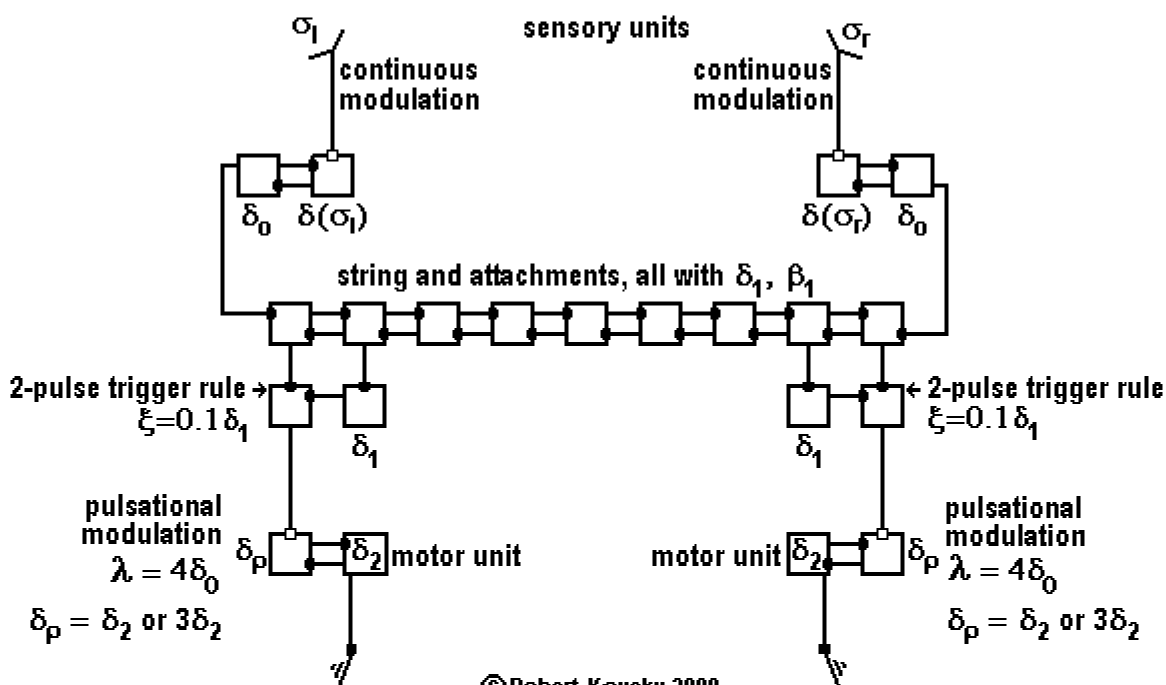


The operating principle of the assembly in Fig. 10 is based on ***balancing*** between input 1 and input 2. When the two pulse streams are equal or balanced, pulses from input 1 will cancel those from input 2 and vice-versa, leading to no output. An imbalance will result in output pulses, pulses from output 1 indicating that input 1 has a higher pulse frequency and pulses from output 2 indicating that input 2 has a higher pulse frequency. Viewed ideally, output 1 should be a pulse train with frequency $(\nu_1 - \nu_2)$ when $\nu_1 > \nu_2$; and output 2 should be a pulse train with frequency $(\nu_2 - \nu_1)$ when $\nu_2 > \nu_1$.

Such idealized operating principle would be the basis for a "frequency subtractor." However, the device assembly of Fig. 10 fails to attain such ideal operations. The "count" deviates from the ideal and has an "error." A modified version presented in the next section corrects the error and (as far as can be seen) achieves idealized operations within its operating range. The modified version, called a "pulse fringer" functions like the "frequency subtractor" should have functioned, but with improvements that lead the way to further developments.

{Fig. 11 shows the frequency subtractor of Fig. 10 in a design for a two-dimensional "engineered organism," adapted from Fig. 27 in *Timing Devices*. The device also incorporates controlling modulation pulses that change the δ of timing devices according to rules discussed in connection with Fig. 4, above. The operating principle of the organism is that two identical systems of "motor units" propel the organism in a watery medium like an ocean through muscular movement of paired appendages. If both motor systems operate at the same rate of drive, the organism will move straight ahead; but, if the rates of drive are different, the organism will turn. Switchable motor units each have two rates of drive, a faster rate where the period is $2\delta_p$ and a slower rate where the period is $4\delta_p$. When there is no output from the string, both motor units drive at the same (faster) rate and the direction is "straight ahead." Pulses from a selector output will reduce the rate of drive of the affected motor unit (for a period denoted by λ), thus "turning the organism" in its watery medium. The controlling pulses through selector outputs are the result of "subtraction" between pulse train inputs with different frequencies, coming from two sensory inputs. The sensory inputs might monitor light or a food molecule. A further development, "Utricle," uses a similar design to detect orientation with respect to gravity. The differences and balances are arranged so that the organism "seeks" out the light or food molecule, balances with respect to gravity, or "follows a gradient" in the intensity of such stimulus.}

Figure 11: a string of coupled timing devices incorporated in an "engineered organism"

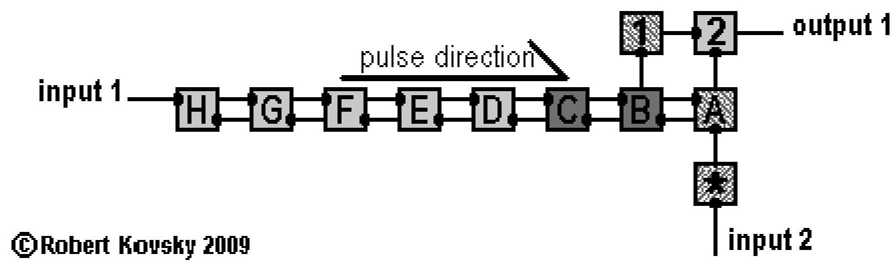


§ 4 Source of error in the frequency subtractor; overcoming error with an advanced device construction: the frequency fringer.

Figure 12 shows the source of error in the frequency subtractor of § 3. A pulse traveling rightwards is being counted by the selector and has put timing device A into the responding condition. "It so happens" that the timing device marked * at input 2 is also in a responding condition. When * discharges, the pulse will not trigger A because A will not be ready. The pulse at * will have no effect on the count. There will be no cancellation of a pulse from input 1.

Under some circumstances, the error can be severe. If the two pulse trains have frequencies that are exactly equal, it is possible for a pulse from input 1 to occupy timing device A on every occasion when input 2 is in the responding condition . Output 1 will reproduce the pulse train from input 1 instead of maintaining silence as a frequency subtractor should.

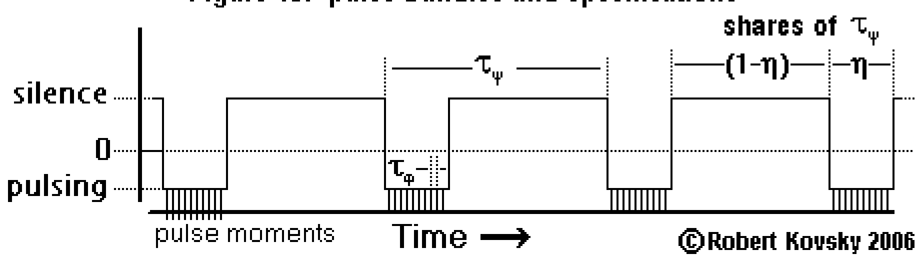
Figure 12: "counting" error in a string of coupled timing devices/selector



The error will not always occur and may not be fatal. If the string of coupled timing devices is very long and the two frequencies are very low, only a few pulses will be in the string at any time, a "traffic jam" at timing device A will occur only rarely and miscounts will be relatively few. Mathematical exactitude is likely unimportant to a rudimentary "engineered organism."

{There are several ways to correct the error. The large-scale way is through development of system-wide principles so that the nature of signals in the system changes from "pulse trains" to "bundles of pulses." Figure 13 shows the nature of pulse bundles.

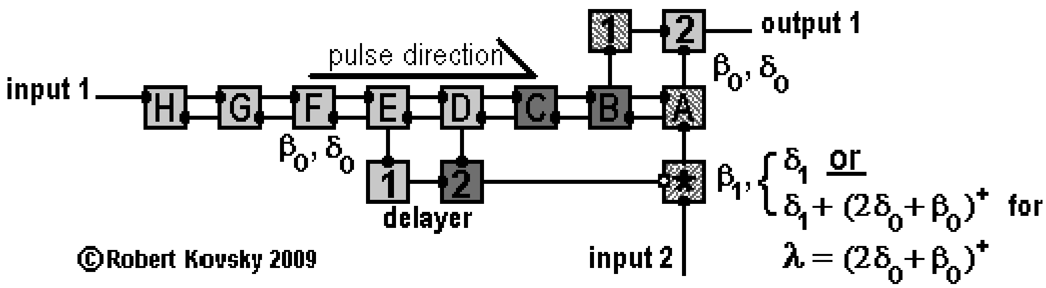
Figure 13: pulse bundles and specifications



Suppose each signal to input 1 and input 2 is a pulse bundle, with τ_φ fixed within each bundle but not between bundles. The period of the two input bundles, τ_ψ, is the same; so is the "active" part of the cycle, η. The bundles differ as to the number of pulses in the bundle. In operations using the device shown in Fig. 10, the two bundles are first incident on the two inputs at the same instant. The entire pulse bundle incident on input 2 completely clears timing device A before the arrival at A of the first pulse from the bundle incident on input 1 that has survived cancellation, thus avoiding the error noted above. And, of course, vice-versa symmetrically. There are wide ranges of values for signals for which such operations are practical.}

In this essay, signals are not pulse bundles, but rather pulse trains (including pulse trains with irregular or slowly varying periods). For pulse trains (and for pulse bundles too), the system in Figure 14 provides an error-free count. The system is counting a pulse directed rightwards while the discharge from timing device * is ***delayed*** until the rightwards-directed pulse clears timing device A and A is again ready. The delay of $(2\delta_0 + \beta_0)^+$ is a "bit more" than sufficient to accomplish this. The pulse from the delayed discharge of timing device * cancels the next pulse that has originated from input 1, maintaining the correct "count."

Figure 14: modified string/selector that counts without error



{Exploring further details, timing intervals β_0 and δ_0 govern all of the timing devices in the assembly in Figure 14 except for timing device * which is governed by β_1 and a δ that alternates between two values, a short δ_1 and a long period that allows the rightwards-directed pulse to clear timing device A before * discharges. The projection from timing device 2 in the "delayer" that becomes a modulatory projection onto timing device * carries a pulse that lengthens the response period, δ, in timing device * from δ_1 to $[\delta_1 + (2\delta_0 + \beta_0)]^+$. The superscript "+" means that "a bit more" is needed to avoid the edge of operations. A longer response period in timing device * lasts for the period of the extension; that is, $\lambda = (2\delta_0 + \beta_0)^+$. However, to attain the largest possible bandwidth (see discussion of τ_{MIN} below), there is a linear attenuation in the lengthening of δ so that the delay fades to 0 at the end of the modulation period. The change in the response period occurs regardless of the condition of timing device *, affecting any response process that is ongoing at the time of arrival or one starting afterwards.

The response process in timing device * must run its course before that timing device will be ready for the next pulse. To accommodate the longest delay at timing device *, the τ_{MIN} for the pulse train incident on input 2 changes from $\tau_{MIN} = [\delta_1 + \beta_1]$ to $\tau_{MIN} = [\delta_1 + (2\delta_0 + \beta_0)^+ + \beta_1]$. Neither δ_1 nor β_1 need be larger than 0^+; and operations in Fig. 14 are premised on both $\delta_1 << \delta_0$ and $\beta_1 << \delta_0$. The change in τ_{MIN} for input 2 is implemented for input 1 also. For the assembly, the ultimate $\tau_{MIN} = (3\delta_0)^+$. If $\delta_0 = 10^{-4}$ sec., the maximum frequency allowable on the assembly is reduced from about 5000 Hz (please see text following Fig. 7) to about 3000 Hz.

The longest period properly handled by the system is denoted τ_{MAX}. τ_{MAX} depends on the number of elements in the string between the output selectors, denoted as "n." $\tau_{MAX} \sim 2n\delta_0$. However τ_{MAX} only limits the higher frequency input pulse train. There is no limit to τ_{MAX} for the lower frequency pulse train; that is, τ_{MAX} for that train can be indefinitely large, even becoming silent.}

Carrying out the modifications on both ends of the string of coupled timing devices, Figure 15 shows a complete “frequency fringer” that performs a mathematical subtraction process.

Figure 15: frequency fringer

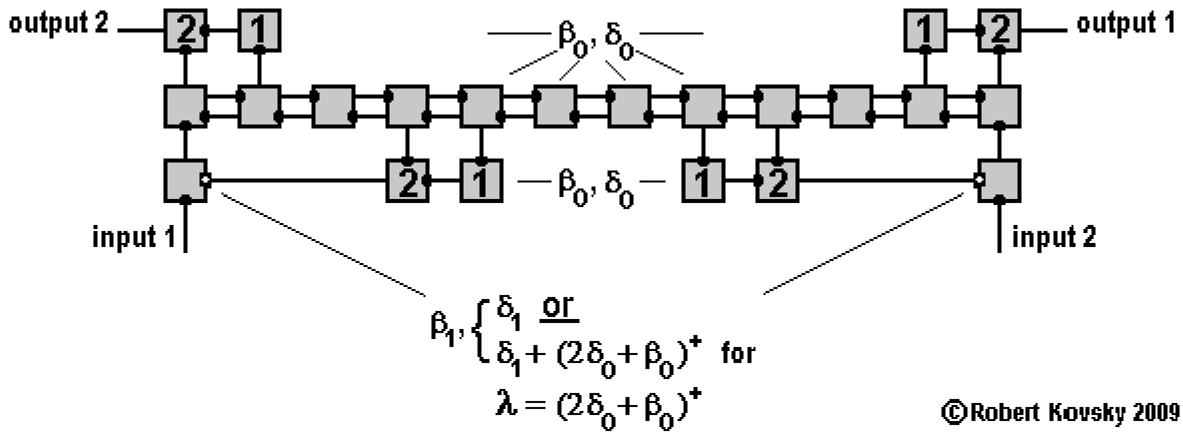
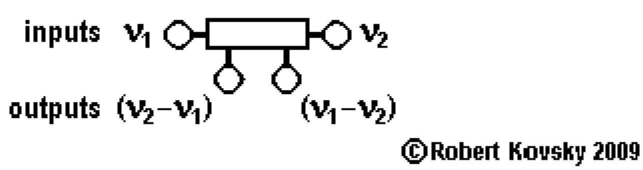


Figure 16 shows a simplified or schematic form of the frequency fringer. For convenience in subsequent constructions, the flow of activity in Figure 16 is from the top to the bottom of the image.

Figure 16: frequency fringer (simplified form)



The name “frequency fringer” is taken from resemblances to “waves of light” in general and to an “interferometer” in particular. An interferometer is an optical device that splits a beam of light into two beams and subjects the beams to different treatments that create an interference pattern when the beams are recombined. The interference pattern consists of a strip of dark blocs alternating with blocs of light. Each coupled pair is a “fringe.” As a situation varies, it can happen that the pattern changes; and a researcher “counts fringes” that appear or disappear.

§ 5 Application of the frequency fringer: "an Ear for Pythagorean Harmonics."

a. ***The Pythagorean Harmonics.*** A striking feature of human perception is the fact that two different tones, heard together, can give rise to a pleasant sensation additional to that felt when the tones are heard separately. Such pleasant sensations, called ***consonance***, are a foundation of music. Another pair of tones might give rise to an unpleasant sensation, called ***dissonance***, which is heard in music as a kind of tension. In simple children's songs, there is a tone called the ***tonic*** and other tones are pleasant or tense according to their relationship to the tonic. The song ends on the tonic, and possibly other consonant tones, that give pleasure, satisfaction and release from tension.

For purposes here, the focus is on well-defined pleasant sensations that are widely experienced, even "universal," for certain pairs of tones. Each such tone is a "pure frequency," called a ***pitch*** and usually notated as a certain number of "cycles per second" or Hertz (Hz). A pure tone of 440 Hz corresponds to the sound of a conventionally-tuned violin playing an open A string. A "pitch pipe" is a whistle-like device that produces accurate pure tones for musicians to tune by.

The ancient sage Pythagoras (6^{th} century B.C.E.) first discovered that consonance can be described by simple ratios of whole numbers, such as 3:2. Expressed in the modern language of frequencies, when the frequencies of two pitches are in a simple whole number ratio, the pair of tones gives rise to pleasant sensations or consonance. When the ratio of frequencies is not simple, the sound is dissonant or tense. A musical scale consists of tones that are in certain ratios with respect to the tonic and with one another. Some ratios identify pleasant pairs and other ratios identify tense pairs. Pleasure and tension are thus identified with ratios (also known as "pitch relations") and ordered accorded to numbers. These concepts organize the body of knowledge known as "harmony" that have been investigated by many scholars. Without commitment to their grander principles or to their institutional proclivities, I concur with many passages in Victor Zuckerkandl's *Sound and Symbol: Music and the External World* (1956) and find utility in the music theory system of Heinrich Schenker.

The fundamental ratio is 1:1 or unison. The most pleasant ratio is 2:1, or the octave. Then, ratios are found to be pleasant in order according to a scheme: 3:2 (dominant); 4:3 (subdominant); 5:4 (mediant), etc. Ratios can be combined, e.g., [(3/2)x(3/2)] = 9:4 (dominant of the dominant).

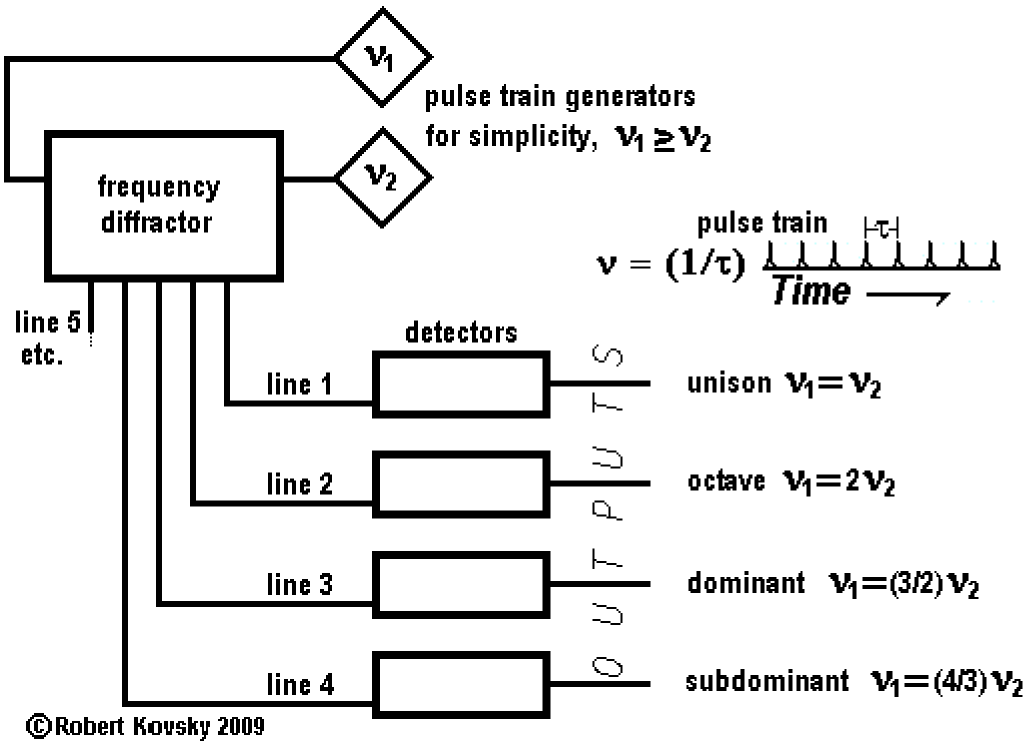
A chief feature of musical experience is that the much the "same" pleasure is aroused by two tones that are in a specific pitch relation, such as 3:2, regardless of the absolute pitch. E.g., two tones of 600 Hz and 400 Hz, heard together, will arouse much the same musical experience as 660 Hz and 440 Hz, heard together. Accordingly, in music, it is possible to "transpose" one set of pitches in one "key" (tonic and subordinate tones) to another set of pitches in another key and yet sustain the musical feelings of pleasure and tension almost exactly.

"An ear for Pythagorean Harmonics," assembled out of timing devices, suggests a basis for these features of perception.

b. ***overall design of "an ear for Pythagorean harmonics."***

As shown in Figure 17, the design of the ear is made up of three components: (1) two sources of input pulse trains, with fixed frequencies of pulse generation, ν_1 and ν_2; (2) a central unit, called the "frequency diffractor;" and (3) "lines" from the frequency diffractor that lead to "detectors," which produce signals on "outputs." Signals are pulse trains (or other closely similar pulse patterns).

Figure 17: an ear for Pythagorean harmonics



The ear functions as follows. The pulse train generators direct two fixed pulse trains onto the two inputs of the frequency diffractor. If the frequencies are the same, a pulse train (with period τ_d) appears on the "unison" output. If $\nu_1 = 2\nu_2$, a signal (with period τ_d) appears on the octave output. If the frequencies have the ratio $\nu_1/\nu_2 = 3:2$, a pulse train (again with period τ_d) appears on the "dominant" output, and so forth. Only a few lines are shown and additional lines identify other simple ratios. If there is no simple ratio between the frequencies, no signal appears on an output. No more than one output signals.

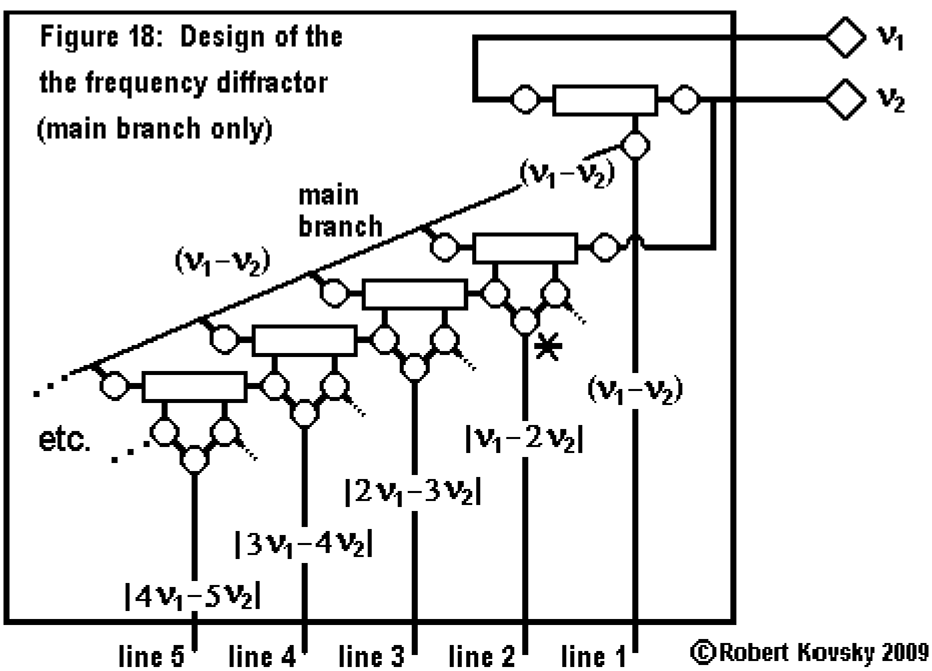
The name "frequency diffractor" continues the optical metaphor. In optical diffraction, interference "fringes" alternately pile up or cancel each other to produce bright spots and shadows. Optical diffraction patterns are typically two-dimensional (and can be three-dimensional), more complex than the one-dimensional pattern of the "fringer."

c. ***detailed design of the ear: from frequency fringer to frequency diffractor***

If there are two inputs to an operating frequency fringer and there is no output from either output branch, the conclusion is that the frequencies of the two input pulse trains are equal. This condition of "silence" is the key to the ear for Pythagorean harmonics. Line 1 exiting from the frequency diffractor is silent when the two tones are in unison; and the detector on line 1 is a device that "turns on" when the input is silence. This key is reproduced in successive stages to generate all the Pythagorean harmonics.

The design of the frequency diffractor, shown in Figure 18, involves multiple frequency fringers. To examine operations, begin from the upper right hand corner, where the first frequency fringer has one output with frequency $(\nu_1 - \nu_2)$. Since $\nu_1 \geq \nu_2$, the other output, never active, is conveniently omitted. The active output, with frequency $(\nu_1 - \nu_2)$, drives both line 1 and the main branch of the frequency diffractor. When that frequency is 0, both line 1 and the main branch are silent; in such case, the two frequencies are in "unison," i.e., $\nu_1 = \nu_2$.

Continuing down the main branch in Figure 18, two frequencies are input into the next frequency fringer in the frequency diffractor, one input pulsing at $(\nu_1 - \nu_2)$ and one input pulsing at ν_2. If $(\nu_1 - \nu_2) > \nu_2$, one output of this device is pulsing at $(\nu_1 - 2\nu_2)$; if $\nu_2 > (\nu_1 - \nu_2)$, the other output is pulsing at $(2\nu_2 - \nu_1)$. Let both of these outputs become inputs to a separate timing device as shown in Figure 5 marked with a * ; regardless of which output is pulsing, the pulsing on line 2 occurs at a rate of $|\nu_1 - 2\nu_2|$, the usual symbols denoting absolute value. If line 2 is silent, $\nu_1 = 2\nu_2$ and the pitches are in the relationship of the "octave."



In the next stage of the main branch, pulses at the rate $(\nu_1 - \nu_2)$ are matched against those at the rate $(2\nu_2 - \nu_1)$, producing output pulsing at the rate of $|2\nu_1 - 3\nu_2|$; line 3 is silent when

$\nu_1 = (3/2)\nu_2$, the case when the two pitches are in the relationship known as the "dominant" in the system of Pythagorean harmonics. The arithmetic goes forward for the successive stages.

Only the "main branch" of the design is shown in Figure 21. Additional frequency fringers can be attached to side branches to handle the various cases. Outputs from various fringers can be interconnected to additional fringers. A large family of "ratios" can be generated.

d. *detailed design of the ear: silence detectors*

The detectors detect ***silence***. E.g., when there is "no pulse train" on line 3 coming out of the frequency diffractor, the "dominant" detector detects and signals that fact by producing a pulse train on its output. If there is silence (no pulse train) on line 3, there are pulse trains on all the other lines, none of the other detectors detects silence and no signal appears on any other output.

Figure 19 shows a design for a silence detector, constructed from a pulse train generator and a "gate timing device" introduced in connection with Figure 5. The pulse train generator in Figure 19 is the kind used before with a frequency ν_d [period $\tau_d = (1/\nu_d)$] as input to a gate timing device.

The input pulse train to the gate timing device produces an identical output pulse train (with a time delay) when "the gate is open;" but there is no output when "the gate is closed." When "the gate is open," each input pulse produces an output pulse after a period δ.

The gate is open or closed depending on activity on the "line." The line controls the gate. The gate is normally open. When a pulse appears on the line, the gate is closed thereafter for a time period T. If the gate is already closed when the next pulse arrives (as illustrated in Figure 19), the closure continues for an additional period T.

Figure 19: Functional design of silence detectors